CLASS XII
SAMPLE PAPER
MATHS

Time allowed – 3 hrs | Max. Marks: 100

General Instructions:-

(i) All questions are compulsory.

(ii) The question paper consists of 29 questions divided into three sections A, B and C. Section A contains 10 questions of 1 mark each, section B comprises of 12 questions of 4 marks each and section C comprises of 7 questions of 6 marks each.

(iii) All questions in section A are to be answered in one word, one sentence or as per the exact requirement of the question.

(iv) There is no overall choice. However, internal choice has been provided in 4 questions of 4 marks each and 2 questions of 6 mark each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculator is not permitted. You may ask for logarithmic tables, if required.

SECTION – A

1. Find the values of x, for which $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

2. If $A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$, then find the value of θ satisfying the equation $A^T + A = I_2$.

3. Construct a 2 x 2 matrix $A = [a_{ij}]$ where elements are given by

$$a_{ij} = \frac{(i + 2j)^2}{2}$$

4. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, find the value of $x + y + xy$.

5. Let A = {1,2,3} and B = {1,2,3,4}. Let there be a relation R = {(x,y): (x,y) Є A x B, y = x + 1 }. Write R in roster form.

6. Discuss the applicability of Rolle's Theorem for the function $f(x) = \tan x$ on $[0, \pi]$.

7. A balloon, which always remains spherical, has a variable diameter $\frac{3}{2}(2x+1)$. Find the rate of change of its volume with respect to x.

8. Evaluate : $\displaystyle\int \frac{1+\cot x}{x+\log(\sin x)}\,dx$

9. Form the differential equation representing the family of curves $y = a\sin(x+b)$, where a and b are arbitrary constants.

10. Find the projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$.

SECTION – B

11. Using properties of determinants, prove that

$$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

12. Let $A = N \times N$ and $*$ be a binary operation on A defined by $(a,b) * (c,d) = (a+c, b+d)$. Show that $*$ is commutative and associative. Find the identity element for $*$ on A, if any.

OR

Show that the relation R on **R** defined as $R = \{(a,b): a \leq b\}$ is reflexive and transitive but not symmetric.

13. Prove that : $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2},\ x \in (0, \frac{\pi}{4})$.

14. If $x = a\cos^3\theta$ and $y = a\sin^3\theta$, find $\frac{d^2y}{dx^2}$.

15. If the function :

$$f(x) = \begin{cases} 3ax+b & \text{if } x > 1 \\ 11 & \text{if } x = 1 \\ 5ax-2b & \text{if } x < 1 \end{cases}$$

is continuous at $x = 1$, find the values of a and b.

16. Find the equation of the tangent to $16x^2+9y^2 = 144$ at (x_1,y_1) where $x_1=2$ and $y_1 > 0$.

17. Find $\int [\sqrt{\cot x} + \sqrt{\tan x}\,]\, dx$.

18. Evaluate : $\int_0^{\frac{\pi}{4}} \log(1+\tan x)\, dx$

OR

Prove that $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$ and hence prove that $\int_0^{\frac{\pi}{2}} \frac{\sin^4 x}{\sin^4 x+\cos^4 x} dx = \frac{\pi}{4}$

19. 3 bad eggs are mixed with 7 good ones. 3 eggs are taken at random from the lot. Find the probability distribution of number of bad eggs drawn.

20. Evaluate : $\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x \cdot dx$

21. Solve the differential equation :

$$(\tan^{-1}y - x)\, dy = (1+y^2)\, dx.$$

OR

Solve the differential equation :

$$x \cos\left(\frac{y}{x}\right) dy = \left\{y \cos\left(\frac{y}{x}\right) + x\right\} dx$$

22. If $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$, find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$ and $\vec{c} \cdot \vec{d} = 15$.

OR

If with reference to right handed system of mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$, then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta_1} + \vec{\beta_2}$ where $\vec{\beta_1}$ is parallel to $\vec{\alpha}$ and $\vec{\beta_2}$ is perpendicular to $\vec{\alpha}$.

SECTION – C

23. If $A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{pmatrix}$, find AB.

Use the result to solve the following system of linear equations:
$2x - y + z = -1$, $-x + 2y - z = 4$, $x - y + 2z = -3$.

24. Find the area of the region enclosed between the two circles $x^2+y^2 = 4$ and $(x-2)^2 + y^2 = 4$.

OR

Evaluate : $\int_0^4 (x+e^{2x})dx$ as limit of sums.

25. An open box, with a square base is to be made out of a given quantity of a metal sheet of area c^2. Show that the maximum volume is $\dfrac{c^3}{6\sqrt{3}}$.

OR

Determine the intervals in which the function
$f(x) = \dfrac{3}{10}x^4 - \dfrac{4}{5}x^3 - 3x^2 + \dfrac{36}{5}x + 11$ is strictly increasing or decreasing.

26. Find the Cartesian as well as vector equations of the plane through the intersection of the planes $\vec{r}.(2\hat{i}+6\hat{j})+12 = 0$ and $\vec{r}.(3\hat{i} - \hat{j}+4\hat{k}) = 0$ which are at a unit distance from the origin.

27. An insurance company insured 2000 scooter drivers, 4000 car drivers and 6000 truck drivers. The probability of accident is 0.01, 0.03 and 0.15 respectively. One of the insured persons meets with an accident. What is the probability that he is a scooter driver?

28. Find the shortest distance between the two lines whose vector equations are $\vec{r} = (3-t)\hat{i} + (4+2t)\hat{j} + (t-2)\hat{k}$ and $\vec{r} = (1+s)\hat{i} + (3s-7)\hat{j} + (2s-2)\hat{k}$, s and t are scalars.

29. If a young man rides his motorcycle at 25 km/hr, he has to spend at Rs. 2 per km on petrol. If he rides at a faster speed of 40 km/hr, the petrol cost increases at Rs. 5 per km. He has Rs. 100 to spend on petrol and wishes to find what is the maximum distance he can travel within one hour. Express this as an LPP and solve it graphically.